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A. Stalder and U. Dürig

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Nanoguitar: Oscillating string as force sensor

A. Stalder and U. Dürig
IBM Research Division, Zurich Research Laboratory, Säumerstr. 4, CH-8003 Rüschlikon, Switzerland

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A force sensor is presented that makes use of an oscillating string. In a conventional scanning force microscopy setup comprising a tip and a sample consisting of a flexible cantilever beam, a string is attached to the free end of the beam along its deflection axis. Changes of the tip–sample interaction force modify the string tension and hence the resonance frequency of transverse oscillations. These oscillations can have amplitudes of the order of microns without causing noticeable wavering of the cantilever beam. This kind of sensor is particularly suitable for applications that require a stiff sensor. A prototype was built using a carbon fiber 5 μm in diameter and 4 mm in length, oscillating at 4 kHz. A force resolution of 2.5 nN was achieved in vacuum for a response time of 1 ms and a sensor stiffness of 160 N/m. For a conventional beam deflection sensor of equal stiffness this corresponds to a deflection sensitivity of 0.015 nm. © *1995 American Institute of Physics.*

I. INTRODUCTION

In scanning force microscopy, forces are usually detected by measuring the deflection of a cantilever beam (CB) that acts as a spring.[1] These CBs typically have spring constants of the order of 1–10 N/m, which makes them unsuitable for situations where large negative force gradients arise because mechanical instabilities can occur. Such conditions exist, for example, in plastic deformation of nanometer-sized metallic contacts. Here negative force gradients of the order of 50 N/m occur. This makes it necessary to use CBs with spring constants of at least 100 N/m. Detection of forces of the order of 1 nN thus requires that CB deflections have to be detected with a precision of a hundredth of a nanometer, which is rather difficult in practice.

We have developed an alternative force sensor that combines high stiffness with high sensitivity and easy detection properties. A conventional setup in scanning force microscopy with a tip and a sample CB is supplemented by a string attached to the free end of the CB (see Fig. 1). A change of the tip–sample interaction force leads to a proportional change of the resonance frequency of the string. The oscillation amplitude of the string can be several microns without causing noticeable wavering of the CB, making detection relatively easy. A different implementation of a sensor based on the same principle has been proposed in Ref. 2.

II. PRINCIPLE OF OPERATION AND PRACTICAL IMPLEMENTATION

In order to determine the properties of the force sensor and to estimate the limits of force resolution, we discuss the physics of an oscillating string using the classical theory of elasticity.[3] First we consider an "ideal" string, i.e., a string that is infinitely hard with respect to stretching and yet infinitely soft with respect to bending. The string is spanned under a tensile force F between two points separated by a distance l. The string is characterized by its mass per unit length τ. The first eigenmode is a sine with a wavelength of $2l$, and the first eigenfrequency is

$$\omega=\frac{\pi}{l}\sqrt{\frac{F}{\tau}}. \tag{1}$$

The force sensitivity becomes

$$\frac{\delta\omega}{\delta F}=\frac{1}{2}\,\frac{\omega}{F}=\frac{\pi}{2l}\,\frac{1}{\sqrt{\tau F}}. \tag{2}$$

In order to maximize the force sensitivity, it is best to lower the tension. This results, however, in a decrease of resonance frequency, which lengthens the response time of the sensor. Although this can be compensated for by making the string lighter or shorter, either approach is limited in the case of a real string. Making a real string lighter ultimately also makes it softer. If the string is shortened, its stiffness with respect to bending can no longer be neglected. Thus in both cases one condition for an ideal string is violated.

Therefore we consider a real string given by its radius r, density ρ, and modulus of elasticity E. Both ends of the string are clamped to a rigid base. A detailed analysis[4] reveals that a real string behaves essentially like an ideal string under a higher tensile force $F+F_0$. This intrinsic tensile force is

$$F_0=E\,\frac{\pi}{4}\,r^4\left(\frac{2\pi}{l}\right)^2, \tag{3}$$

i.e., it depends only on the elasticity and the dimensions of the string. Equations (1) and (2) for resonance frequency and force sensitivity are approximately valid for a real string if F is replaced by $F^*=F+F_0$. For low tensile forces of the order of some F_0 or smaller, it is more accurate to multiply the resonance frequency by a factor of $\sqrt{4/3}$ and to change Eq. (2) accordingly. Theoretically the force sensitivity is maximal if the fiber is operated under pressure. In practice however, a slight bending of the string is difficult to avoid, which hampers the exertion of significant pressure.

In a prototype force sensor we used a carbon fiber 5 μm in diameter[5] and 4 mm long as the oscillating string. The fiber has a modulus of elasticity $E=29.4\times10^{10}$ N/m^2 and a mass per length of $\tau=3.71\times10^{-8}$ kg/m. The intrinsic tensile force becomes $F_0=25$ μN and the resonance frequency with

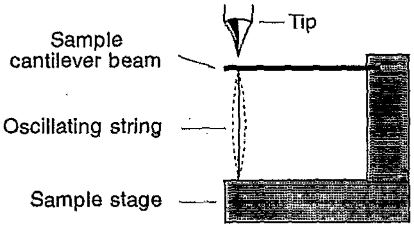

FIG. 1. Schematic setup of the string force sensor. The interaction force between tip and sample cantilever beam modifies the string tension and hence the resonance frequency of the string. Detection of the resonance frequency is facilitated by the fact that the oscillation amplitude of the string can be of the order of microns without causing noticeable wavering of the cantilever beam. From Ref. 4. Reprinted by permission of Kluwer Academic Publishers.

no applied tension is expected to be 3.7 kHz. Typically, the fiber was operated under slight tension at a frequency of 4.2 kHz.

The force resolution of the fiber is limited by thermal noise in the string oscillation. For a harmonic oscillator the frequency noise is given by[6]

$$\left(\frac{\delta\omega}{\omega}\right)^2=\frac{kT}{2QE_{\mathrm{osc}}}\frac{\Delta\omega}{\omega}. \tag{4}$$

Here $\Delta\omega$ denotes the bandwidth of the measurement, Q the quality factor of the resonance, E_{osc} the oscillation energy, T temperature, and k Boltzmann's constant. Using the approximation of an ideal string, the oscillation energy becomes

$$E_{\mathrm{osc}}=F^*\Delta l, \tag{5}$$

where

$$\Delta l=\frac{\pi^2}{4}\frac{A^2}{l} \tag{6}$$

is the extent to which the fiber is stretched during oscillation with amplitude A. The thermal limit for force detection becomes

$$\delta F_{\min}=\sqrt{2kT\frac{F^*}{\Delta l}\frac{\Delta\omega/\omega}{Q}}. \tag{7}$$

It is again preferable to lower the tension without decreasing the resonance frequency as not to lengthen response times. Another way to improve the force resolution is to increase the oscillation amplitude; however, this is limited by nonlinear effects at large amplitudes. A more detailed discussion of force resolution will follow below.

The typical oscillation amplitude of the carbon fiber was 4 μm, which yields $\Delta l=10$ nm. We observed a quality factor of $Q=1000$ in vacuum and 10 in air. This yields a theoretically attainable force resolution of $2.5\,\mathrm{pN}/\sqrt{\mathrm{Hz}}$ in vacuum and $25\,\mathrm{pN}/\sqrt{\mathrm{Hz}}$ in air.

In the actual sensor one end of the fiber is attached to the end of a flexible CB, which is also the sample. For optimal sensitivity this CB should be relatively soft because the tip–sample interaction force is split between the CB and fiber, which act as two parallel springs. Therefore the fiber feels only a portion of a change in the interaction force ΔF_{int}, namely

$$\Delta F=\Delta F_{\mathrm{int}}\frac{C_f}{C_f+C_{\mathrm{cb}}}, \tag{8}$$

where C_{cb} is the transversal spring constant of the CB and

$$C_f=E\frac{\pi r^2}{l} \tag{9}$$

is the longitudinal spring constant of the fiber.

We chose a CB having $C_{\mathrm{cb}}=110$ N/m. For the carbon fiber we expect $C_f=1500$ N/m. Experimentally such a high spring constant could only be achieved at relatively high tensions (resonance frequency close to 10 kHz) because the fiber had to be straightened first. At the typical operation frequency of 4.2 kHz the spring constant of the fiber was about 50 N/m, yielding a total stiffness of the sensor of 160 N/m.

Another important design criterion for the CB is that the gap between tip and sample should not be disturbed by the oscillation of the fiber. On the other hand a large oscillation amplitude of the fiber is desired for optimal force resolution. We resolved this dilemma by choosing a CB that has a lower resonance frequency than the fiber. Thus, owing to the slow response time of the CB, even large fiber oscillations will result in only minor CB oscillations. In this case the periodic force on the CB is $C_f(\Delta l/2)\cos 2\omega t$. The amplitude of the CB oscillation becomes

$$A_{\mathrm{cb}}=\frac{\Delta l}{2}\frac{C_f}{C_{\mathrm{cb}}}\left(\frac{\omega_{\mathrm{cb}}}{2\omega}\right)^2, \tag{10}$$

where $(\omega_{\mathrm{cb}}/2\omega)^2$ is the transfer function of the CB in the approximation of a harmonic spring with a resonance frequency of $\omega_{\mathrm{cb}}\ll 2\omega$ and a quality factor of $Q_{\mathrm{cb}}\gg 2\omega/\omega_{\mathrm{cb}}$.

Our prototype CB has a resonance frequency of 900 Hz and a quality factor of about 100, so that both of the above conditions are fulfilled and the CB oscillation amplitude of about 0.02 nm is negligible. Care has to be taken not to make the resonance frequency of the CB too small, which would limit the response time of the sensor. We have used a phase-locked-loop device of the kind described in Ref. 6 to detect the resonance frequency of the fiber. As it takes approximately four oscillation periods for this device to determine the oscillation frequency anyway, the sensor is not slowed down by our choice of the CB resonance frequency.

Even when the oscillation amplitude of the CB is small, the CB can be statically deflected to a considerable extent because the oscillating fiber pulls on it until an equilibrium between pushing and pulling is reached. In the limit of a very hard fiber $C_f\gg C_{\mathrm{cb}}$, this CB deflection is $\Delta l/2$. Otherwise it is given by the following force balance:

$$C_f\left(\frac{\Delta l}{2}-\Delta l_{\mathrm{cb}}\right)=C_{\mathrm{cb}}\Delta l_{\mathrm{cb}},$$

which yields

$$\Delta l_{\mathrm{cb}}=\frac{\Delta l}{2}\frac{C_f}{C_f+C_{\mathrm{cb}}}. \tag{11}$$

At typical operating conditions the CB was deflected by 1.5 nm.

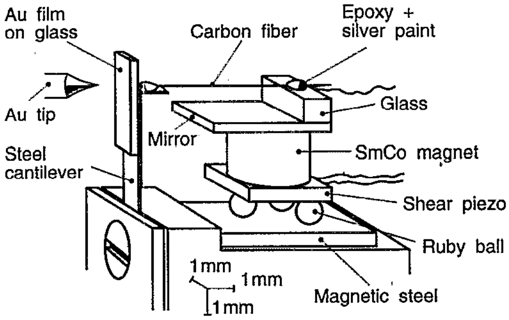

FIG. 2. Drawing of the prototype force sensor. A 5-μm-diam carbon fiber is used as oscillating string. The fibre is glued to a steel cantilever beam and a friction glider, which serves to adjust the fiber tension. The fiber oscillation is excited using the Lorentz force by sending an ac current through the fiber, which is immersed in the field of a permanent magnet. The motion of the fiber is detected optically by focusing a laser beam onto the fiber and detecting the bypassing light. From Ref. 4. Reprinted by permission of Kluwer Academic Publishers.

Figure 2 is a drawing of the prototype force sensor. The carbon fiber is attached with low-vapor pressure epoxy[7] to a steel CB and a friction glider, which serves to adjust the fiber tension. The ruby balls stick and slide on the magnetic steel plate when an electrical sawtooth signal is applied to the shear piezo. The magnet provides the necessary friction and helps avoid fast glider motion at the steep edge of the sawtooth signal by means of eddy current damping.

In addition, the magnetic field, which is perpendicular to the mirror surface, serves to excite the fiber resonance. For this purpose, an ac current is sent through the fiber so that it feels the Lorentz force perpendicular to both the fiber and the magnetic field. The motion of the fiber is detected optically. A laser beam is focused onto the fiber; the bypassing light is reflected from the mirror and detected in a photodiode.

The setup in Fig. 2 is part of a scanning tunneling microscope for use in vacuum. The Au tip is attached to a piezo scanner (not visible in Fig. 2). The sample consists of an Au film evaporated epitaxially onto a glass plate. This setup was used to investigate the electrical and mechanical properties of atomic-sized Au contacts.[8]

The piezo glider can be moved in steps of 200 nm, which results in changes of the resonance frequency of about 200 Hz per step (at the typical operation frequency of 4.2 kHz). A means to adjust the fiber tension is necessary because after gluing the fiber, the tension is not well defined. Further we found that when the fiber was put into vacuum, its resonance frequency rose so markedly that it became impossible to excite the oscillation sufficiently for detection. Therefore *in situ* adjustment of the tension was indispensable. We suspect that this increase of frequency is connected with the evaporation of water in the fiber because after flooding the vacuum chamber with nitrogen and evacuating it again, the effect was less pronounced.

Using the glider, the force sensor can be easily and reliably calibrated *in situ*. For this purpose the shear piezo is displaced a given distance, Δz_g, which deflects the CB by Δz_{cb}, stretches the fiber, and thus changes its resonance frequency. In this situation the fiber and the CB act as springs in series. Therefore both feel the same force, namely

$$\Delta F = C_f(\Delta z_g - \Delta z_{cb}) = C_{cb}\Delta z_{cb}. \tag{12}$$

Measurement of the corresponding change of resonance frequency $\Delta\nu$ yields the sensitivity of the fiber to changes of the tensile force. During measurement of the tip–sample interaction forces, however, only the portion (8) of the interaction force is felt by the fiber. Therefore, the force sensitivity becomes

$$\frac{\delta\nu}{\delta F_{\text{int}}} = \frac{\Delta\nu}{\Delta F}\,\frac{\Delta F}{\Delta F_{\text{int}}} = \frac{\Delta\nu}{C_{cb}\Delta z_g}. \tag{13}$$

The spring constant of the CB can be determined *ex situ*; we used a microbalance for this purpose. The calibration of the glider movement with respect to applied voltage was done *ex situ* using an optical position sensor as described in Ref. 9. It is important for practical purposes that the force sensitivity can be determined without knowledge of the longitudinal spring constant C_f of the fiber. In our test setup we were able to determine C_f by measuring the CB deflection Δz_{cb} using the tunnel current between the tip and the sample, and *then* using Eq. (12). Usually, however, this possibility is not available in a scanning force microscope.

Experimentally we achieve a force sensitivity of 0.01 Hz/nN, which is a factor of 2 less than what would be expected using the approximate formula (2) with F replaced by $F+F_0$ and taking Eq. (8) into account. The typical frequency noise in vacuum is 25 mHz for a response time of 1 ms, which yields an experimental force resolution of 2.5 nN. The theoretical limit for the interaction force resolution obtained using Eqs. (7) and (8) is 0.1 nN.

III. IMPROVEMENTS

Finally, we shall consider how the force resolution can be improved. The natural way to do this is to lower the intrinsic tension F_0, because it would be difficult to build a sensor that can be used under pressure. We keep the CB as it is and change the dimensions of the fiber. In order not to lengthen the response time, we keep the resonance frequency of the fiber constant. It turns out that both radius and length of the fiber have to decrease, and that consequently its longitudinal spring constant decreases too. The scaling laws are

$$r \sim F_0^{1/3}, \quad l \sim F_0^{1/6}, \quad C_f \sim F_0^{1/2}.$$

Furthermore, we demand that the amplitude A_{cb} of the CB oscillation remain constant, which implies that the amplitude A of the fiber oscillation has to increase. The minimal interaction force that can be detected scales then like $(C_f + C_{cb})C_f^{1/2}$. Thus it decreases more slowly as soon as C_f is smaller than C_{cb}. However the force resolution is more stringently limited for the following reason.

So far, it has been assumed that the amplitude of the fiber oscillation is relatively small. Hence the energy contained in stretching and compressing the string during oscillation, namely

$$E_{el} = \frac{1}{2}\,C_f\left(\frac{\Delta l}{2}\right)^2,$$

can be neglected compared to E_{osc} given in Eq. (5). If this is not the case, a nonlinear term that depends on the amplitude of the fiber oscillation has to be added to the equation of motion of the string. This modified equation of motion can lead to whirling motions of the fiber, i.e., to motions where the fiber "pops out" of the plane spanned by the fiber axis and the direction of the excitation. The whirling motions can be periodic, quasiperiodic, or chaotic.[10]

These nonlinearities seem negligible for the prototype we have built, because the ratio between E_{osc} and E_{el} is of the order of 10^3. However if the intrinsic tension of the fiber is lowered as described above, this ratio decreases quickly, namely proportional to C_f^{-2}. Practically, it should be possible to lower this ratio to 10. This corresponds to a carbon fiber with a diameter of 1 μm, a length of 2 mm, and a longitudinal spring constant of 150 N/m. Under the assumption that the effective longitudinal spring constant of the fiber is at least 50 N/m at a resonance frequency of 10 kHz, a force resolution of 0.25 nN could be achieved in vacuum for a response time of 1 ms and an oscillation amplitude of 4.5 μm (corresponding to $\Delta l_{\mathrm{cb}}=3.5$ nm and $A_{\mathrm{cb}}=0.01$ nm).

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